



KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

MATHEMATICS AUGUST PRE PREPARATORY PAPER 2

MARKING GUIDELINE

GEOMETRY	
S	A mark for the correct statement. (A statement mark is independent of a reason)
R	A mark for a correct reason. (A reason mark may only be awarded if the statement is correct)
S/R	Award a mark if the statement AND reason are both correct.

QUESTION 1

No.	SOLUTION	MARK JUSTIFICATION	MA RK																								
1.1	45 children	✓ A answer	(1)																								
1.2	$\bar{X} = \frac{\sum fx}{n}$ $\bar{X} = \frac{(4 \times 2) + (8 \times 10) + (12 \times 9) + (16 \times 7) + (20 \times 8) + (24 \times 7) + (28 \times 2)}{45}$ $\bar{X} = \frac{692}{45}$ $\bar{X} = 15,38 \text{ minutes}$ Answer only: full marks	✓ A 692 ✓ CA answer	(2)																								
1.3	<table><tr><th>Time taken (<i>t</i>) (in minutes)</th><th>Number of children</th><th>Cumulative frequency</th></tr><tr><td>$2 < t \leq 6$</td><td>2</td><td>2</td></tr><tr><td>$6 < t \leq 10$</td><td>10</td><td>12</td></tr><tr><td>$10 < t \leq 14$</td><td>9</td><td>21</td></tr><tr><td>$14 < t \leq 18$</td><td>7</td><td>28</td></tr><tr><td>$18 < t \leq 22$</td><td>8</td><td>36</td></tr><tr><td>$22 < t \leq 26$</td><td>7</td><td>43</td></tr><tr><td>$26 < t \leq 30$</td><td>2</td><td>45</td></tr></table>	Time taken (<i>t</i>) (in minutes)	Number of children	Cumulative frequency	$2 < t \leq 6$	2	2	$6 < t \leq 10$	10	12	$10 < t \leq 14$	9	21	$14 < t \leq 18$	7	28	$18 < t \leq 22$	8	36	$22 < t \leq 26$	7	43	$26 < t \leq 30$	2	45	✓ A first 4 cum freq correct ✓ A last 3 cum freq correct	(2)
Time taken (<i>t</i>) (in minutes)	Number of children	Cumulative frequency																									
$2 < t \leq 6$	2	2																									
$6 < t \leq 10$	10	12																									
$10 < t \leq 14$	9	21																									
$14 < t \leq 18$	7	28																									
$18 < t \leq 22$	8	36																									
$22 < t \leq 26$	7	43																									
$26 < t \leq 30$	2	45																									

1.4	CUMULATIVE FREQUENCY GRAPH (OGIVE) The graph shows a smooth curve on a grid. The x-axis is labeled 'Time in minutes' and ranges from 0 to 30. The y-axis is labeled 'Cumulative Frequency' and ranges from 0 to 50. A horizontal dashed line is drawn at y = 22.5, and a vertical dashed line is drawn from the intersection point down to x = 15 on the x-axis.	✓ CA plotting cum freq at upper limits correctly (all points) ✓ A shape (smooth) ✓ A grounding (2;0)	(3)
1.5	On graph at the y-value of 22,5 or 23 Median = ±15 minutes. Answer only: full marks	✓ CA graph ✓ CA answer	(2)
			[10]
QUESTION 2			
2.1	Answer only: full marks $a = 12,44$ $b = 0,98$ $y = 12,44 + 0,98x$	✓ A value of a ✓ A value of b ✓ CA equation	(3)
2.2.1	Percentage = $\frac{15}{50} \times 100$ = 30%	✓ A answer	(1)
2.2.2	$y = 12,44 + 0,98x$ $y = 12,44 + 0,98(30)$ $y = 41,84$ = 42 OR $y = 41,87$(if using calculator) $y = 42$ OR $y = \frac{21}{50}$ Answer only: full marks	✓ A substitution ✓ CA answer as integer ✓ CA value of y ✓ CA answer as integer ✓✓ CA CA answer	(2)

2.3.1	standard deviation = 13,88	✓✓ A A answer	(2)
2.3.2	$x = 50,67 - 45,67$ $= 5\%$ Answer only: full marks	✓ A 50,67 – 45,67 ✓ A answer	(2)
			[10]
QUESTION 3			
			(1)
3.1	$4 = k(-3) + 3$ $k = -\frac{1}{3}$	A✓substitution	(1)
3.2	$-\frac{1}{3}x + 3 = 0$ $-\frac{1}{3}x = -3$ $x = 9$ Answer only: full marks $\therefore B(9; 0)$	A✓equating to 0 A✓ $x = 9$	(2)
3.3	$E(0; -2)$ $\frac{x+3}{2} = 0$ and $\frac{y+4}{2} = -2$ $x = 3$ and $y = -8$ $C(3; -8)$	A✓ $E(0; -2)$ A✓ $\frac{x+3}{2} = 0$ CA✓ $\frac{y+4}{2} = -2$ CA✓ $C(3; -8)$	(4)
3.4	$m_{BC} = \frac{-8-0}{3-9} = \frac{4}{3}$	CA✓ m_{BC}	(3)

	$m_T = m_{BC}$ (parallel lines) $y = \frac{4}{3}x + c$ $-2 = \frac{4}{3}(15) + c$ $c = -22$ $\therefore y = \frac{4}{3}x - 22$	CA✓ $-2 = \frac{4}{3}(15) + c$ CA✓ $y = \frac{4}{3}x - 22$	
3.5	$\tan \alpha = -2$ $\alpha = 180^\circ - \tan^{-1}(2)$ $\alpha = 116,57^\circ$ $\tan \hat{A}Bx = \tan\left(-\frac{1}{3}\right)$ $\hat{A}Bx = 180^\circ - \tan^{-1}\left(\frac{1}{3}\right)$ $\hat{A}Bx = 161,57^\circ$ $\therefore \hat{B}AC = 161,57^\circ - 116,57^\circ$ (ext $\angle$ of a Δ) $= 45^\circ$	A✓ $\tan \alpha = -2$ A✓ $\alpha = 116,57^\circ$ A✓ $\tan \hat{A}Bx = \tan\left(-\frac{1}{3}\right)$ CA✓ $\hat{A}Bx = 161,57^\circ$ CA✓ $\hat{B}AC = 45^\circ$	(5)
3.6	$AD = \sqrt{(3-(-1))^2 + (4-0)^2} = 2\sqrt{2}$ $AB = \sqrt{(-3-9)^2 + (4-0)^2} = 4\sqrt{10}$ $AT = \sqrt{(-3-15)^2 + (4-(-2))^2} = 6\sqrt{10}$ $\frac{\text{Area of } \Delta ABD}{\text{Area of } \Delta ATC} = \frac{\frac{1}{2} \cdot AD \cdot AB \sin \hat{A}}{\frac{1}{2} \cdot AC \cdot AT \sin \hat{A}}$ $= \frac{AD \cdot AB}{AC \cdot AT}$ $= \frac{(2\sqrt{2})(4\sqrt{10})}{(8\sqrt{10})(6\sqrt{10})}$ $= \frac{\sqrt{5}}{30}$	A✓ AD and AB A✓ AT A✓ $\frac{\frac{1}{2} \cdot AD \cdot AB \sin \hat{A}}{\frac{1}{2} \cdot AC \cdot AT \sin \hat{A}}$ CA✓ $\frac{(2\sqrt{2})(4\sqrt{10})}{(8\sqrt{10})(6\sqrt{10})}$ CA✓ $\frac{\sqrt{5}}{30}$	(5)

			[20]
QUESTION 4			
4.1	$(x-2)^2 + (y-3)^2 = r^2$ $(-1-2)^2 + (-1-3)^2 = r^2$ $9+16 = r^2$ $r^2 = 25$ $\therefore (x-2)^2 + (y-3)^2 = 25$ OR $AN = \sqrt{(-1-2)^2 + (-1-3)^2}$ $AN = \sqrt{9+16}$ $r = 5$ $\therefore r^2 = 25$ $\therefore (x-2)^2 + (y-3)^2 = 25$	A✓subs of N and A into the distance formula A✓ $r^2 = 25$ CA✓equation (3)	
4.2	C(2+5 ; 3) (by symmetry) C(7 ; 3)	A✓ $x = 7$ A✓ $y = 3$	(2)
4.3	$m_{AN} = \frac{3-(-1)}{2-(-1)} = \frac{4}{3}$ $m_{AB} = -\frac{3}{4} \quad (\text{radius} \perp \text{tangent})$ $y - (-1) = -\frac{3}{4}(x - (-1))$ $y = -\frac{3}{4}x - \frac{3}{4} - 1$ $y = -\frac{3}{4}x - \frac{7}{4}$	A✓subs A and N into gradient formula A✓ $m_{\text{radius}} = \frac{4}{3}$ A✓ $m_{\text{tangent}} = -\frac{3}{4}$ A✓subs A and m CA✓equation (5)	

	OR $m_{AN} = \frac{3 - (-1)}{2 - (-1)} = \frac{4}{3}$ $m_{AB} = -\frac{3}{4} \quad (\text{radius} \perp \text{tangent})$ $y = -\frac{3}{4}x + c$ $-1 = -\frac{3}{4}(-1) + c$ $c = -\frac{7}{4}$ $\therefore y = -\frac{3}{4}x - \frac{7}{4}$	A✓subs A and N into gradient formula A✓ $m_{\text{radius}} = \frac{4}{3}$ A✓ $m_{\text{tangent}} = -\frac{3}{4}$ A✓subs A and m CA✓equation	(5)
4.4	B(7 ; y_B) $y_B = -\frac{3}{4}(7) - \frac{7}{4}$ $y_B = -7$ B(7 ; -7) BC = 10 units	A✓sub $x = 7$ A✓ $y_B = -7$ CA✓BC = 10	(3)
4.5	$(x+1)^2 + (y+1)^2 = 1$	A✓LHS A✓RHS	(2)
4.6	$d_c = \sqrt{(x_1 + x_2)^2 + (y_1 + y_2)^2}$ N(2 ; 3) M(6 ; -5) $MN = \sqrt{(2+6)^2 + (3+(-5))^2}$ $= \sqrt{68}$ $= 8,25$ $r_1 + r_2 = 5 + 9 = 14$ $d_c < r_1 + r_2$ $\therefore \text{The circles intersect}$	A✓ $MN = \sqrt{(2+6)^2 + (3+(-5))^2}$ A✓ 8,25 A✓ 14 CA✓ $d_c < r_1 + r_2$ CA✓conclusion	(5)
			[20]
QUESTION 5			
5.1.1	$\sin(34^\circ + 30^\circ)$ $\cos 34^\circ \cos 30^\circ - \sin 34^\circ \sin 30^\circ$ $\frac{\sqrt{3}}{2} \cos 34^\circ - \frac{1}{2} \sin 34^\circ$ $\frac{\sqrt{3}}{2} p - \frac{1}{2} \sqrt{1 - p^2}$	✓ A expansion ✓ A special angles ✓ A simplification	(3)

5.1.2	$\cos 68^\circ = 2 \cos^2 34^\circ - 1$ $= 2p^2 - 1$	✓ A expansion ✓ A Answer	(2)
5.1.3	$\cos 34^\circ = 1 - 2 \sin^2 17^\circ$ $p = 1 - 2 \sin^2 17^\circ$ $\sqrt{\frac{1-p}{2}} = \sin 17^\circ$	✓ A half angle ✓ A substitution ✓ A answer	(3)
5.1.4	$2 \sin^2 28^\circ - 1 + 1$ $(-1 + 2 \sin^2 28^\circ) + 1$ $-(1 - 2 \sin^2 28^\circ) + 1$ $-\cos 56^\circ + 1$ $-\sqrt{1-p^2} + 1$	✓ A Expansion ✓ A simplification ✓ A Answer	(3)
5.2.1	$\frac{\sin 70^\circ \cdot \tan 60^\circ}{\cos 180^\circ \tan 70^\circ \sin 20^\circ}$ $\frac{\sin 70^\circ \sqrt{3}}{(-1) \frac{\sin 70^\circ}{\cos 70^\circ} \cdot \cos 70^\circ}$ $-\sqrt{3}$	✓ A $\sin 70^\circ$ ✓ A $\cos 180^\circ$ ✓ A $\tan 70^\circ$ ✓ A $\sin 20^\circ$ ✓ A $\frac{\sin 70^\circ}{\cos 70^\circ}$ ✓ A $\sin 20^\circ = \cos 70^\circ$ ✓ CA $-\sqrt{3}$	(7)
5.2.2	$1 - 2 \sin^2 22,5^\circ$ $\cos 2(22,5^\circ)$ $\cos 45^\circ$ $\frac{1}{\sqrt{2}}$	✓ A Expansion ✓ A Simplification ✓ CA $\cos 45^\circ$ ✓ CA Answer	(4)
5.3.1	$\sin^2 x = 0$ $\sin x = 0$ $x = 0^\circ$ or $x = 180^\circ$ or $x = 90^\circ$	✓ A $x = 0^\circ$ ✓ A $x = 180^\circ$ ✓ A $x = 90^\circ$	(3)

5.3.2	$\frac{\cos 2x \cdot \tan x}{\sin^2 x}$ $\frac{(\cos^2 x - \sin^2 x) \left(\frac{\sin x}{\cos x} \right)}{\sin^2 x}$ $\frac{\cos^2 x - \sin^2 x}{\sin x \cos x}$ $\frac{\cos^2 x}{\sin x \cos x} - \frac{\sin^2 x}{\sin x \cos x}$ $\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}$ $\frac{\cos x}{\sin x} - \tan x = \text{RHS}$	✓ A Expansion ✓ A $\frac{\sin x}{\cos x}$ ✓ A Simplification ✓ A simplification ✓ A Answer	(5)
			[30]
QUESTION 6			
6.1	$a = 3$ and $b = 2$	✓ A $a = 3$ ✓ A $b = 2$	(2)
6.2	Period = 360°	✓ A 360°	(1)
6.3	$y \in [2; 4]$	✓ A Values 2 and 4 ✓ A Notation	(2)
6.4	$0^\circ < x < 45^\circ$ or $90^\circ < x < 135^\circ$	✓ A 0° and 45° ✓ A 90° and 135° ✓ A Notation	(3)
6.5	$y = \cos 2x$ $y = \sin(90^\circ + 2x)$ $y = \sin 2(x + 45^\circ)$ $q = 45^\circ$	✓ A $y = \cos 2x$ ✓ A co ratio ✓ A 45°	(3)
			[11]
QUESTION 7			
7.1	In $\triangle PMQ$: $\tan \theta = \frac{x}{QM}$ $QM = \frac{x}{\tan \theta}$	✓ A Trig ratio ✓ A Answer	(2)

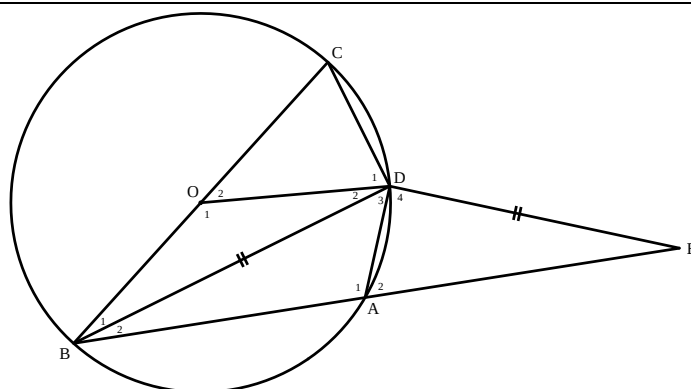
7.2	$\Delta PMQ \equiv \Delta PMS$ [AAS / RHS] $MR = \frac{x}{\tan \theta} = QM$ $\hat{QMR} = 180^\circ - 2\beta$ $\sin \beta \times \frac{\tan \theta}{x} = \frac{\sin(180^\circ - 2\beta)}{12x}$ $\tan \theta = \frac{\sin 2\beta}{12x} \times \frac{x}{\sin \beta}$ $\tan \theta = \frac{\cos \beta}{6}$	✓ A MR=QM ✓ A Correct substitution ✓ A Reduction ✓ A Double angle	(4)
7.3	$\frac{x}{QM} = \frac{\cos \beta}{6}$ $x = \frac{60 \cos 40^\circ}{6}$ $x = 7,66$ The height of the lighthouse is 8 metres	✓ A Equating ✓ A Subst. QM=60 and $\beta = 40^\circ$ ✓ A Answer	(3)
			[09]

QUESTION 8

8.1.1	$\hat{R} = 109^\circ$ [opp $\angle$ s of a cyclic quad]	A✓S A✓R	(2)
8.1.2	$\hat{P}_1 + 71^\circ = 90^\circ$ [$\angle$ in a semicircle] $\hat{P}_1 = 19^\circ$	A✓S/R A✓Answer	(2)
8.1.3	$\hat{O}_1 = 2 \times 19^\circ$ [$\angle$ at centre = 2 x $\angle$ at circumference] $= 38^\circ$	A✓S A✓R	(2)

8.2.1	$\hat{S}\hat{T}R = 90^\circ$ [∠ in a semicircle] $\therefore$ TSOM is a cyclic quad [converse opp ∠s of a cyclic quad]	A✓S/R A✓S/R	(2)
8.2.2	$ON \perp PT$ [line from centre to midpoint of chord] $\therefore PT \parallel OM$ [co-int ∠s supplementary/ corresp ∠=]	A✓S A✓R A✓R	(3)
8.2.3	$\hat{M}_1 = \hat{T}_2$ [corresp ∠s, $PT \parallel OM$] $\hat{T}_2 = \hat{S}_1$ [∠s in the same segment] $\therefore \hat{M}_1 = \hat{S}_1$	A✓S A✓R A✓S A✓R	(4)
			[15]
9.1	Constr. Let M and N lie on AB and AC respectively such that $AM = DE$ and $AN = DF$. Draw MN In $\triangle AMN$ and $\triangle DEF$ $AM = DE$ [constr...] $AN = DF$ [constr...] $\hat{A} = \hat{D}$ [given] $\therefore \triangle AMN = \triangle DEF$ [SAS]	A✓constr. A✓S/R A✓S	(6)

	$\therefore \hat{AMN} = \hat{E} = \hat{B} \quad [\Delta s]$ $\therefore MN \parallel BC \quad [\text{corresp } \angle s \text{ are equal}]$ $\frac{AB}{AM} = \frac{AC}{AN} \quad [\text{line } \parallel \text{ one side of } \Delta / \text{ prop theorem, } MN \parallel BC]$ but $AM = DE$ and $AN = DF$ $\therefore \frac{AB}{DE} = \frac{AC}{DF}$	A✓S/R A✓S A✓R	
9.2	In $\triangle ACG$ $\frac{AE}{EC} = \frac{AF}{FG} \quad [\text{line } \parallel \text{ one side of } \Delta / \text{ prop theorem, } EF \parallel CG]$ $\frac{3p}{2p} = \frac{2k}{FG}$ $FG = \frac{4k}{3}$ In $\triangle BFD$ $\frac{BG}{GF} = \frac{BC}{CD} \quad [\text{line } \parallel \text{ one side of } \Delta / \text{ prop theorem, } DF \parallel CG]$ $\frac{\left(\frac{11k}{3}\right)}{\left(\frac{4k}{3}\right)} = \frac{BC}{CD}$ $\frac{BC}{CD} = \frac{11}{4}$	A✓S/R A✓S A✓S/R A✓S A✓Answer	(5)
			[11]

QUESTION 10

10.1	$\hat{CDB} = 90^\circ$ [$\angle$ in a semicircle]	A✓S A✓R	(2)
10.2	Let $\hat{B}_1 = x$ $\hat{B}_2 = \hat{B}_1$ [given] $\hat{C} = 90^\circ - x$ [sum of $\angle$ s of $\triangle BCD$] $\hat{A}_2 = \hat{C} = 90^\circ - x$ [ext $\angle$ of a cyclic quad ABCD] $\hat{E} = \hat{B}_2 = x$ [$\angle$ s opp = sides] $\hat{D}_4 = 180^\circ - \hat{A}_2 - \hat{E}$ [sum of $\angle$ s of $\triangle$] $= 180^\circ - (90^\circ - x) - x$ $= 90^\circ$	A✓ $\hat{C} = 90^\circ - x$ A✓ $\hat{A}_2 = \hat{C} = 90^\circ - x$ A✓ $\hat{E} = \hat{B}_2 = x$ A✓ $\hat{D}_4 = 180^\circ - \hat{A}_2 - \hat{E}$ A✓ Answer	(5)
10.3	In $\triangle BDO$ and $\triangle BED$ $\hat{B}_1 = \hat{B}_2$ [given] $\hat{D}_2 = \hat{B}_1$ [$\angle$ s opp = sides] $\therefore \hat{D}_2 = \hat{E}$ [both = x] $\hat{O}_1 = \hat{BDE}$ [3rd $\angle$] $\therefore \triangle BDO \parallel \triangle BED$ [AAA]	A✓ $\hat{D}_2 = \hat{B}_1$ A✓ $\hat{D}_2 = \hat{E}$ A✓R	(3)
10.4	$\frac{BD}{BE} = \frac{OB}{BD}$ [$\parallel$ $\triangle$ s] $BD^2 = OB \cdot BE$ but $BD = DE$ and $OB = \frac{1}{2} BC$ $\therefore DE^2 = \frac{1}{2} BC \cdot BE$ $2DE^2 = BC \cdot BE$	A✓S A✓R A✓ $BD^2 = OB \cdot BE$ A✓S	(4)
			[14]